# MA/MSCMT-06

December - Examination 2016

## M.A./ M.Sc. (Final) Mathematics Examination

## Analysis and Advanced Calculus

## Paper - MA/MSCMT-06

**Time : 3 Hours ]** **[ Max. Marks :- 80**

**Note:** The question paper is divided into three sections A, B and C. Use of non-programmable scientific calculator is allowed in this paper.

**Section - A** **$8 \times 2 = 16$**

(Very Short Answer Questions)

**Note:** Section 'A' contain (08) Very Short Answer Type Questions. Examinees have to attempt **all** questions. Each question is of 02 marks and maximum word limit may be thirty words.

1) (i) Write uniqueness theorem of differential equations for Banach space.

(ii) Define regulated function for Banach space.

(iii) Define orthogonal projection for Hilbert spaces.

(iv) Define complete orthonormal set for Hilbert space.

(v) Define inner product space.

(vi) Define first dual space.

(vii) Define multilinear mapping.

(viii) Explain weak convergence.

**Section - B** **$4 \times 8 = 32$**

(Short Answer Questions)

**Note:** Section 'B' contain 08 Short Answer Type Questions. Examinees have to delimit each answer in maximum 200 words.

2) State and prove Holder's inequality for normed linear space.

3) State and prove Reisz lemma for normed linear space.

4) Prove that a closed convert subset K of Hilbert space H contains a unique vector of smallest norm.

5) Prove that if M and N are closed linear subspaces of Hilbert space H such that M $\perp$ N then then the linear subspaces M + N is closed.

6) Prove that every Hilbert space is reflexive.

7) Prove that if T is normal operator on a Hilbert space H then eigen spaces of T are pair wise orthogonal.

8) If X and Y are two banach spaces over the same field K of scalers and V is an open subset of X. Let $f : V \rightarrow Y$ be continuous functions. Let $u$, $v$ be any two distinct points of V such that $[u, v] \subset v$ and if f is differentiable in $[u, v]$ then prove that. $|| f(v) - f(u) || \leq || v - u || \sup \{|| Df(x) || \, | x \in [u, v]\}$

9) State and prove Global uniqueness theorem.

## Section - C **$2 \times 16 = 32$**

(Long Answer Questions)

**Note:** Section 'C' contain 04 Long Answer Type Questions. Examinees will have to answer any two (02) questions Each questions is of 16 marks. Examinees have to delimit each answer in maximum 500 words.

10) State and prove closed graph theorem for normed linear space.

11) State and prove Hahn-Banach theorem for normed linear spaces.

12) State and prove implicit function theorem on differentiate functions over Banach sapces.

13) If f be a function on a compact interval $[a, b]$ of R into a Banach space X over K. then prove that f is regulated if and only if:

    (i) For each point $C \in [a, b]$

    $$\lim_{\substack{t \to c \\ t > c}} f(t) \text{ exists}$$

    (ii) For each point $C \in [a, b]$

    $$\lim_{\substack{t \to c \\ t < c}} f(t) \text{ exists}$$